ELSEVIER

Nuclear Instruments and Methods in Physics Research A 445 (2000) 105–109

NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

www.elsevier.nl/locate/nima

Effects of bunch density gradient in high-gain free-electron lasers☆

Zhirong Huang*, Kwang-Je Kim

Advanced Photon Source, Argonne National Laboratory, 9700 S. Cass Avenue, Argonne, IL 60439, USA

Abstract

We investigate effects of the bunch density gradient in self-amplified spontaneous emission, including the role of coherent spontaneous emission (CSE) in the evolution of the free-electron laser process. In the exponential gain regime, we solve the coupled Maxwell–Vlasov equations and extend the linear theory to a bunched beam with energy spread. A time-dependent, nonlinear simulation algorithm is used to study the CSE effect and the nonlinear evolution of the radiation pulse. Published by Elsevier Science B.V.

PACS: 41.60.Cr; 42.25.Lc

Keywords: Bunched beam; Coherent spontaneous emission; High-gain free-electron laser; Self-amplified spontaneous emission

1. Introduction

The one-dimensional (1D) theory of self-amplified spontaneous emission (SASE) is based on the solution of the linearized Maxwell–Vlasov equations, for the cases of a coasting beam with energy spread [1] and a bunched monochromatic beam [2]. However, most SASE demonstration experiments operate at relatively long wavelengths but employ short, intense electron bunches. Effects of bunched beams have been considered in Refs. [3–8] for simple bunch profiles. Spiking behavior near the bunch tail was observed in the nonlinear simulation [3,6]. In this paper, we extend the linear theory to bunched beams with arbitrary phase-space distributions and evaluate the effects of the bunch density gradient in the exponential gain regime. A time-dependent, nonlinear simulation is developed to take into account the coherent spontaneous emission (CSE) and to study the nonlinear evolution of the radiation pulse.

2. Linear analysis

For FEL interaction, the backward wave is dropped from the Maxwell equation [8], and the slowly varying envelope approximation (SVEA) is invoked for the transverse electric field amplitude:

$$A(z,t) = E(z,t)e^{ick_f(ct-z)} \tag{1}$$

where $ck_f = ck_w\beta_{||}/(1-\beta_{||})$ is the forward resonant frequency, ck_w is the wiggler frequency, and $c\beta_{||}$ is the longitudinal bunch velocity.

☆Work supported by the US Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.

*Corresponding author. Tel.: +630-252-6023; fax: +630-252-5703.

E-mail address: zrh@aps.anl.gov (Z. Huang).

0168-9002/00/$ - see front matter. Published by Elsevier Science B.V.
PII: S0168-9002(00)00122-4

It is convenient to define the bunch coordinate as $\theta = k_f(z - ct) + k_w z$ and change the independent variables from (z, t) to (z, θ). The phase-space distribution of the electron bunch is given by the Klimontovich distribution [1]

$$F(\theta, \eta, z) = \frac{k_f}{n_0} \sum_{j=1}^{N} \delta(\theta - \theta_j(z))\, \delta(\eta - \eta_j(z)) \tag{2}$$

where n_0 is the maximum line density and $\eta = (\gamma - \gamma_0)/\gamma_0$ is the conjugate variable to θ. Hence, the Maxwell equation becomes

$$\left(\frac{\partial}{\partial z} + k_w \frac{\partial}{\partial \theta}\right) A = \kappa_1 n_0 \mathrm{e}^{-\mathrm{i}\theta} \int \mathrm{d}\eta\, F(\theta, \eta, z) \tag{3}$$

where $\kappa_1 = ec^2 \mu_0 a_w / (2\sqrt{2}\gamma_0 \Sigma)$. Here μ_0 is the permeability of free space, Σ is the beam transverse cross section, a_w is equal to the wiggler parameter K for a helical wiggler, and $a_w = K[JJ]/\sqrt{2}$ for a planar wiggler. The Vlasov equation for the electron distribution is [1,2]

$$\frac{\partial F}{\partial z} + 2k_w \eta \frac{\partial F}{\partial \theta} - \kappa_2 A \mathrm{e}^{\mathrm{i}\theta} \frac{\partial F}{\partial \eta} = 0 \tag{4}$$

where $\kappa_2 = ea_w / (\sqrt{2}\gamma_0^2 mc^2)$ is another constant.

In the exponential gain regime and without an external field, we can regard the field amplitude A in Eq. (4) as a small, first-order quantity. This includes the coherent and the incoherent spontaneous emissions as well as the stimulated emission. Hence, we approximate the distribution function F as $F_0 + F_1$. The zeroth-order term F_0 is the initial smooth distribution

$$F_0(\theta, \eta, z) = \chi(\theta - 2k_w \eta z) V(\eta) \tag{5}$$

where $\chi(\theta)$ is the bunch profile $(0 \le \chi \le 1)$ and $V(\eta)$ is the initial energy distribution of the beam $(\int \mathrm{d}\eta\, V(\eta) = 1)$. The first-order term F_1 contains both the initial fluctuation ΔF_0 and the bunching behavior through FEL interaction. Approximating F with F_0 in the third term of Eq. (4) yields

$$F_1 = \Delta F_0 + \kappa_2 \frac{\partial F_0}{\partial \eta} \int_0^z \mathrm{d}s\, A(\theta_0, s)\, \mathrm{e}^{\mathrm{i}\theta_0} \tag{6}$$

where $\theta_0 = \theta - 2k_w \eta z + 2k_w \eta s$. Since the FEL gain becomes negligible when the width of $V(\eta)$ is much larger than the Pierce parameter ρ [1] (defined through $\kappa_1 n_0 \kappa_2 = 4k_w^2 \rho^3$), we have $2k_w \eta z \sim 2k_w \rho z < 2\pi$ in the exponential gain regime. We can therefore make the approximation $\theta_0 \approx \theta$ in the slowly varying amplitude A but keep the fast oscillatory phase $\mathrm{e}^{\mathrm{i}\theta_0}$. Inserting Eqs. (5) and (6) into Eq. (3) and applying the Laplace transformation, we obtain

$$A(\theta, z) = \frac{\kappa_1 k_f}{k_w} \sum_{j}^{\theta_j < \theta} \mathrm{e}^{-\mathrm{i}\theta_j} G(\theta, \theta_j, z) \tag{7}$$

where the Green function is

$$G(\theta, \theta_j, z) = \int \frac{\mathrm{d}\lambda}{2\pi \mathrm{i}} \mathrm{e}^{2\mathrm{i}\lambda[k_w z - (\theta - \theta_j)]} \int \mathrm{d}\eta \frac{V(\eta)}{\lambda - \eta} \times \exp\left[-2\mathrm{i}\rho^3 \int \mathrm{d}\eta \frac{\mathrm{d}V/\mathrm{d}\eta}{\lambda - \eta} \int_{\theta_j}^{\theta} \chi(\theta')\, \mathrm{d}\theta'\right]. \tag{8}$$

The λ-integration is along a straight path parallel to the real axis and below all singularities of the integrand. The Green function is nonzero only when $0 \le (\theta - \theta_j) \le k_w z$ (the slippage length). Hence the total electric field at θ is the sum of fields that originated from the discrete radiators prior to θ but within the slippage length. For a monochromatic beam with $V(\eta) = \delta(\eta)$, Eqs. (7) and (8) reproduce the result of Refs. [2,7]. The Green function can be evaluated asymptotically for a flat-top bunch profile [4], with a $(\theta - \theta_j)$ dependence that resembles Fig. 1, and with a maximum value given by

$$\frac{G_0}{\sqrt{k_w z}} \exp\left(\frac{z}{2L_g}\right) \tag{9}$$

where L_g is the power gain length, and G_0 is a constant.

The intensity of the SASE radiation is

$$I = \frac{1}{2c\mu_0} \langle A(\theta, z) A^*(\theta, z) \rangle = \frac{\kappa_1^2 k_f n_0}{2c\mu_0 k_w^2} \left[\int_{\theta - k_w z}^{\theta} \mathrm{d}\theta'\, \chi(\theta') |G(\theta, \theta', z)|^2 + \frac{n_0 \lambda_r}{2\pi} \left| \int_{\theta - k_w z}^{\theta} \mathrm{d}\theta'\, \mathrm{e}^{-\mathrm{i}\theta'} \chi(\theta') G(\theta, \theta', z) \right|^2 \right] \tag{10}$$

where $\lambda_r = 2\pi/k_f$ is the resonant wavelength. The first term in the square bracket is the usual incoherent SASE due to the shot noise, and the second term is the coherent SASE, growing from the initial coherent spontaneous emission. Coherent bunching only at the resonant wavelength was considered in Ref. [5], and coherent SASE for a monochromatic, bunched beam was calculated in Ref. [7]. Here the result is generalized to bunched beams with arbitrary phase-space distributions.

Integration by parts on the second term of Eq. (10) leads to

$$I_{\mathrm{coh}} = \frac{\kappa_1^2 k_f n_0}{2c\mu_0 k_w^2} \frac{n_0 \lambda_r}{2\pi} \left| \int_{\theta - k_w z}^{\theta} \mathrm{d}\theta' \, \mathrm{e}^{-\mathrm{i}\theta'} \frac{\mathrm{d}(\chi G)}{\mathrm{d}\theta'} - [\mathrm{e}^{-\mathrm{i}\theta'} \chi(\theta') G(\theta, \theta', z)] \Big|_{\theta - k_w z}^{\theta} \right|^2. \tag{11}$$

The second term is negligible because the Green function is exponentially small at the boundary compared with its maximum value. Eq. (11) clearly shows that the coherent SASE comes from the bunch density gradient. Consider a flat-top bunch that has a bunch length θ_b longer than the slippage length $k_w z$. In the slippage region when $0 \le \theta < k_w z$, we have $\mathrm{d}\chi/\mathrm{d}\theta = \delta(\theta)$, and the bunch radiates coherently with an effective point charge $e n_0 \lambda_r/(2\pi)$ and an intensity profile

$$I_{\mathrm{coh}} = \frac{\kappa_1^2 n_0^2}{2c\mu_0 k_w^2} |G(\theta, z)|^2. \tag{12}$$

In the body of the bunch when $k_w z < \theta < \theta_b$, the coherent SASE goes to zero, and the incoherent SASE intensity can be estimated as

$$I_{\mathrm{incoh}} \approx \frac{\kappa_1^2 k_f n_0}{2c\mu_0 k_w^2} \frac{G_0^2}{2} \exp\left(\frac{z}{L_g}\right). \tag{13}$$

Thus, the ratio of the maximum coherent SASE intensity to the incoherent one is

$$\frac{I_{\mathrm{coh}}^{\max}}{I_{\mathrm{incoh}}} \approx \frac{n_0 \lambda_r}{\pi k_w z} \tag{14}$$

which can be a very large number for long-wavelength SASE experiments. Nevertheless, the spike of the coherent SASE will be less intense for a more realistic bunch profile.

Following Ref. [7], one can write Eq. (7) as

$$A(z, \theta) = \frac{\kappa_1 k_f}{2k_w} \sum_j^{\theta_j < \theta} \mathrm{e}^{-\mathrm{i}\theta_j} \int_{-\infty + \mathrm{i}\varepsilon}^{\infty + \mathrm{i}\varepsilon} \frac{\mathrm{d}\nu}{2\pi\mathrm{i}} \mathrm{e}^{-\mathrm{i}\nu(\theta - \theta_j)} \times \int \frac{\mathrm{d}\lambda}{2\pi\mathrm{i}} \frac{\mathrm{e}^{2\mathrm{i}\lambda k_w z}}{D(\lambda, \nu, \theta, \theta_j)} \int \mathrm{d}\eta \, \frac{V(\eta)}{\lambda - \eta} \tag{15}$$

where ε is an infinitesimal and positive number,

$$D = \lambda - \frac{\nu}{2} + \rho^3 w(\theta, \theta_j) \int \mathrm{d}\eta \frac{\mathrm{d}V/\mathrm{d}\eta}{\lambda - \eta} \tag{16}$$

and

$$w(\theta, \theta_j) = \frac{1}{(\theta - \theta_j)} \int_{\theta_j}^{\theta} \chi(\theta') \, \mathrm{d}\theta' \le 1. \tag{17}$$

For the coasting beam, $w(\theta, \theta') = 1$ and $D(\lambda, \nu) = 0$ is the dispersion relation including the energy spread [1]. Eq. (16) provides a generalization to the bunched beam. When the bunch distribution does not change appreciably over the slippage length, $w(\theta, \theta_j) \approx \chi(\theta)$ from Eq. (17), and the FEL gain is affected only by the local electron density as expected.

3. Nonlinear simulation

For numerical simulation of bunch density gradient effects, it is convenient to use the individual particle formulation of FEL equations [6]

$$\frac{\partial \theta_j}{\partial \bar{z}} = \bar{\eta}_j \tag{18}$$

$$\frac{\partial \bar{\eta}_j}{\partial \bar{z}} = -[a \mathrm{e}^{\mathrm{i}\theta} + \mathrm{c.c.}] \tag{19}$$

$$\left[\frac{\partial}{\partial \bar{z}} + \frac{1}{2\rho} \frac{\partial}{\partial \theta}\right] a = \frac{k_f}{n_0} \sum_{j=1}^{N} \mathrm{e}^{-\mathrm{i}\theta_j} \delta(\theta - \theta_j) \tag{20}$$

where $\bar{z} = 2k_w \rho z$, $\bar{\eta} = \eta/\rho$, and $a = 2\rho k_w A/(\kappa_1 n_0)$ is the scaled electric field. The partial derivative with respect to θ in Eq. (20) describes the slippage between the electron bunch and the radiation field. The sum over the δ functions can be approximated by a local average over a small bin $\Delta\theta$

around θ, i.e.,

$$\int_{\theta-\Delta\theta/2}^{\theta+\Delta\theta/2} \frac{d\theta}{\Delta\theta} \left[\frac{k_f}{n_0} \sum_{j=1}^{N} e^{-i\theta_j} \delta(\theta-\theta_j) \right] = \frac{\chi(\theta)}{N_\theta} \sum_{j \in N_\theta} e^{-i\theta_j} \tag{21}$$

where $N_\theta = \chi(\theta) n_0 \Delta\theta / k_f$ is the number of electrons within a $\Delta\theta$ bin at position θ.

A time-dependent simulation algorithm [3] can be constructed to take into account the slippage effect: one first divides the bunch into many buckets (separated by λ_r) and loads each bucket with simulation particles that are uniformly distributed in $\Delta\theta = 2\pi$ and have the proper energy spread. Apply Eqs. (18)–(20) without the slippage term in each bucket, and then slip the computed field one bucket forward after each wiggler period. To start up the FEL process, one either gives a small, constant bunching [3] or uses the shot noise simulation algorithm of Ref. [9]. However, such a discretization is not adequate for CSE simulation because the bunch profile $\chi(\theta)$ is only sampled with a sampling interval λ_r. Thus, the Fourier transform of $\chi(\theta)$ is defined only between the Nyquist critical frequency $f_c = c/(2\lambda_r)$ or $\omega_c = ck_f/2$, and the coherent bunching around the resonant frequency ck_f is left out.

We modify the above algorithm to include the CSE effect by decreasing the sampling interval to cover the resonant part of the bunch spectrum. For example, after loading every bucket with the prescribed simulation particles, we can further divide each bucket into eight sections with $\Delta\theta = \pi/4$ so that the critical sampling frequency is $4ck_f$. The spectral power of the bunch profile outside this frequency range should be sufficiently small to eliminate the effect of aliasing. In each section, Eq. (21) is used to determine the average bunching, and the electric field is computed and propagated section by section just as in the time-dependent algorithm. The final electric field at the exit of the wiggler is averaged over the resonant wavelength (or eight sections), in consistent with the slowly varying envelope approximation.

Compared with the multifrequency approach to CSE simulation [6], this time-dependent approach is more straightforward and can include the shot noise in a natural way. For example, let us take

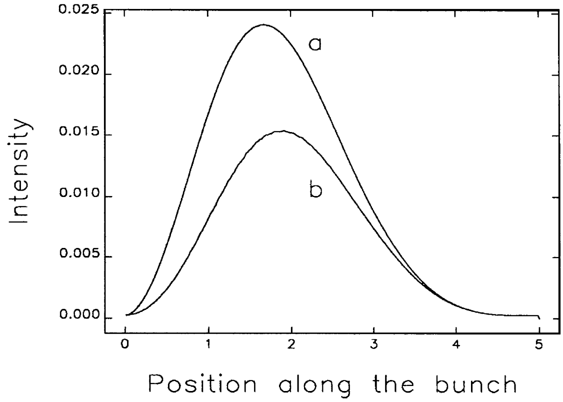

Fig. 1. Coherent SASE intensity $|a_{coh}|^2$ versus $2\rho\theta$ for a flat-top bunch (a) without initial energy spread, (b) with a flat-top energy spread of width ρ, at $\bar{z} = 5$.

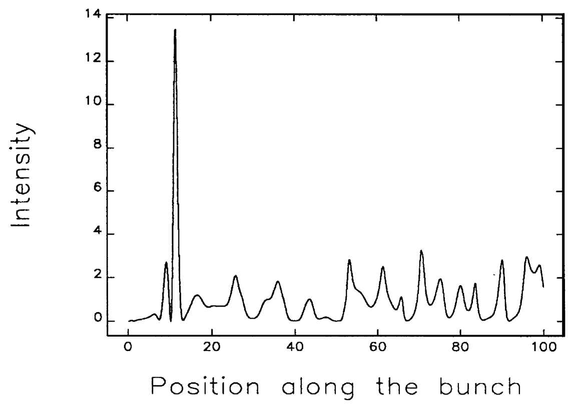

Fig. 2. Coherent and incoherent SASE intensity $|a|^2$ versus $2\rho\theta$ for a flat-top bunch ($0 < 2\rho\theta < 100$) without initial energy spread at $\bar{z} = 15$.

$N = 10^{10}$, $\rho = 1/(40\pi)$, and a flat-top bunch with the bunch length equal to $1000\lambda_r$. Fig. 1 shows the coherent SASE at the slippage region of the bunch (i.e., $0 < 2\rho\theta < \bar{z}$), in agreement with Eq. (12) of the linear theory. Fig. 2 shows the total intensity profile in the nonlinear regime after saturation. In addition to the spikes seeded by the noise nonuniformity in the entire radiation pulse [4], a more intense spike seeded by CSE exists at the slippage region, in agreement with the simulation of Ref. [6]. We emphasise that the CSE spike exists both in the linear and the nonlinear regimes (as in Figs. 1 and 2) and is different from the superradiant spike observed in the nonlinear regime [3], which originates from the discontinuity of the initial condition used in the simulation.

4. Conclusions

A linear theory and a nonlinear simulation algorithm are developed to treat SASE for bunched beams with arbitrary phase-space distributions. In general, sharp density variation over a radiation wavelength in the electron bunch gives rise to the coherent spontaneous emission, which in turn drives the coherent amplified emission within the slippage distance.

References

[1] K.-J. Kim, Nucl. Instr. and Meth. A 250 (1986) 396.
[2] J.-M. Wang, L.-H. Yu, Nucl. Instr. and Meth. A 250 (1986) 484.
[3] R. Bonifacio et al., Phys. Rev. A 40 (1989) 4467.
[4] R. Bonifacio et al., Phys. Rev. Lett. 73 (1994) 70.
[5] K.-J. Kim, S.J. Hahn, Nucl. Instr. and Meth. A 358 (1995) 93.
[6] N. Piovella, AIP Conf. Proc. 413 (1997) 205.
[7] S. Krinsky, Phys. Rev. E 59 (1999) 1171.
[8] Z. Huang, K.-J. Kim, in: Proceedings of the 1999 Particle Accelerator Conference.
[9] C. Penman, B.W.J. McNeil, Opt. Commun. 90 (1992) 82.